2.2.3 Curve Plotting-II

▶ Here we will plot some cartesian graphs for build up our concept. You will try all of these graphs and try to understand the key ideas behind them.

▶ Curve Type-I: $y = 0,\ y = a,\ y = -a,\ y = x,\ y = |x|,\ y = x^2,\ y = x^3,\ y = x^4,\ y = -x,\ y = -|x|,\ y = -x^2,\ y = -x^3,\ y = -x^4,\ y = \sqrt{x},\ y = -\sqrt{x},\ y = \sqrt{x+2},\ y = \sqrt{x-2},\ y = \frac{2}{x},\ y = \frac{3}{x},\ y = \frac{3}{x} + 1,\ y = \frac{3}{x} - 1,\ y = \frac{3}{x-1},\ y = \frac{3}{x+1}$.

▶ Curve Type-II: $y = [x],\ y = -[x],\ y = [x^2],\ y = [x]^2,\ y = [x+2],\ y = |[x]|,\ y = [|x|],\ y = x - [x],\ y = [x+1],\ y = e^x,\ y = e^{-x},\ y = -e^x,\ y = -e^{-x},\ y = \ln x,\ y = |\ln x|$.

▶ Curve Type-III: $y = \sin x,\ y = \cos x,\ y = \tan x,\ y = \sec x,\ y = \csc x,\ y = \cot x,\ y = |\sin x|,\ y = |\cos x|,\ y = |\tan x|,\ y = |\sec x|,\ y = |\csc x|,\ y = |\cot x|$.

▶ Curve Type-IV: $x + y = a,\ x^2 + y^2 = a^2,\ x^3 + y^3 = a^3,\ x^4 + y^4 = a^4,\ x^2 + y^2 = 0,\ |x| + |y| = a,\ y = \sqrt{2 - x^2},\ y = 1 + \sqrt{2 - x^2},\ y = 1 - \sqrt{2 - x^2},\ y = \sqrt{4 - (x-1)^2},\ y = \sqrt{4 - (x-2)^2},\ y = \sqrt{9 - (x-3)^2}$.

▶ Curve Type-V: $y^2 = x$ *vs.* $y = \sqrt{x}$, $y = \sqrt{2 - x^2}$ *vs.* $y^2 + x^2 = 2$, $y = \sqrt{4 - (x-2)^2}$ *vs.* $y^2 + (x-2)^2 = 4$, $y = x(x-1)$ *vs.* $y = x(x-1)(x-2)$ *vs.* $y = x(x-1)(x-2)(x-3)$.

▶ Draw the Regions: *i)* $y = \pi/2,\ y = x,\ y - axis$; *ii)* $0 < x < 1,\ y > 0,\ 1 < x + y < 2$; *iii)* $y \geq 0,\ y \leq x,\ x^2 + y^2 = 1,\ x^2 + y^2 = 2$; *iv)* $0 \leq x, y \leq 1,\ \frac{3}{4} \leq x + y \leq \frac{3}{2}$; *v)* $x, y \geq 0,\ \sqrt{4 - (x-2)^2} \leq y \leq \sqrt{9 - (x-3)^2}$.

▶ Find Range of x, y, z: *i)* $x, y, z \geq 0,\ x^2 + y^2 = 4,\ z = 2,\ x + y = 4$; *ii)* $x + y + z \leq 3,\ y^2 \leq 4x,\ 0 \leq x \leq 1,\ y \geq 0,\ z \geq 0$; *iii)* $z = y^2,\ z = 1,\ x = 0,\ x = 1,\ y = -1,\ y = 1$; *iv)* $x^2 + y^2 + z^2 \leq 1,\ z = 1/2$; *v)* $x = 0,\ y = 0,\ z = 0,\ 6x + 4y + 3z = 12$; *vi)* $x = 0,\ y = 0,\ z = 0,\ z = 1,\ x^2 + y^2 = 1,\ x \geq 0,\ y \geq 0$, *vii)* $xy - plane$ *bounded by* $y = 2 - x^2,\ y = x$ and *upper* z *is bounded by* $z = x + 2$.

2.2.4 Computing Area

▶ Rule 1: Let $y = f(x)$ be a continuous function on $[a, b] \Rightarrow$ Area of the curve enclosed by $x = a$ to $x = b$ is $\int_a^b f(x)dx = \int_a^b y dx$.

▶ Rule 2: Let $x = g(y)$ be a continuous function on $[c, d] \Rightarrow$ Area of the curve enclosed by $y = c$ to $y = d$ is $\int_c^d g(y)dy = \int_c^d xdy$.

[Do It Yourself] 2.8. *Find the area bounded between two parabolas* $y = x^2 + 4$ *and* $y = -x^2 + 6$.

[Do It Yourself] 2.9. *Let* $g\colon [0, 2] \to \mathbb{R}$ *be defined by* $g(x) = \int_0^x (x - t)e^t dt$. *Then area between the curve* $y = g''(x)$ *and the* $x - axis$ *over the interval* $[0, 2]$ *is*
(A) $e^2 - 1$ *(B)* $2(e^2 - 1)$ *(C)* $4(e^2 - 1)$. *(D)* $8(e^2 - 1)$.

[Do It Yourself] 2.10. *Find the area of the region in the first quadrant enclosed by the curves* $y = 0,\ y = x$ *and* $y = \frac{2}{x} - 1$.

[Do It Yourself] 2.11. *The area of the region bounded by* $y = 8$ *and* $y = |x^2 - 1|$ *is*
(A) 50/3 *(B)* 100/3 *(C)* 110/3. *(D)* 52/3.

[Do It Yourself] 2.12. *Find the area of the smaller of the two regions enclosed between* $\frac{x^2}{9} + \frac{y^2}{2} = 1$ *and* $y^2 = x$.

[Do It Yourself] 2.13. *Find the area of the region bounded by* $y = x^3,\ x+y-2 = 0,\ y = 0$.

[Do It Yourself] 2.14. *Find the area of the region bounded by* $y = x^2,\ x + y = 2$.

[Do It Yourself] 2.15. *Find the area of the region bounded by* $y = (x - 2)^2,\ y = 4 - x^2$.

[Do It Yourself] 2.16. *Show that the area bounded by* $x^2+y^2 = 64a^2$ *and* $y^2 = 12ax$ $(a > 0)$ *lying in the positive side of x-axis is* $\frac{16a^2}{3}(4\pi + \sqrt{3})$.

2.2.5 Computing Area in Polar Coordinate

▶ Rule 1: Let $r = f(\theta)$ be a continuous function on $[\theta_1, \theta_2] \Rightarrow$ Area of the curve enclosed by $\theta = \theta_1$ to $\theta = \theta_2$ is $\frac{1}{2}\int_{\theta_1}^{\theta_2} r^2 d\theta$.

▶ Draw all these curve given below and try to find the area.

[Do It Yourself] 2.25. *Find the area of the circle* $r = 2a\sin\theta$.
$[Hint:\ area = \frac{1}{2}\int_0^\pi r^2 d\theta]$

[Do It Yourself] 2.26. *Find the area of the cardioide* $r = a(1 - \cos\theta)$.
$[Hint:\ area = 2 \times \frac{1}{2}\int_0^\pi r^2 d\theta]$

[Do It Yourself] 2.27. *Show that the entire area of the lemniscate* $r^2 = a^2\cos 2\theta$ *is* a^2.

[Do It Yourself] 2.28. *Show that the entire area of* $r = a\cos 2\theta$ *is* $\frac{\pi a^2}{2}$.

[Do It Yourself] 2.29. *Find the entire area of i)* $r = a\sin 2\theta$, *ii)* $r = a\cos 3\theta$, *iii)* $r = a\cos 4\theta$.